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# Universality and quantized response in bosonic mesoscopic tunneling

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 (Dated: July 2015)

We show that tunneling involving bosonic wires and/or boson integer quantum Hall (bIQH) edges is characterized by features that are far more universal than those in their fermionic counterpart. Considering a pair of minimal geometries, we examine the tunneling conductance as a function of energy (e.g., chemical potential bias) at high and low energy limits, finding a low energy enhancement and a universal high versus zero energy relation that hold for all wire/bIQH edge combinations. Beyond this universality present in all the different topological (bIQH-edge) and nontopological (wire) setups, we also discover a number of features distinguishing the topological bIQH edges, which include a current imbalance to chemical potential bias ratio that is quantized despite the lack of conductance quantization in the bIQH edges themselves. The predicted phenomena require only initial states to be thermal and thus are well suited for tests with ultracold bosons forming wires and bIQH states. For the latter, we highlight a potential realization based on single component bosons in the recently observed Harper-Hofstadter bandstructure.

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## I. INTRODUCTION

Tunneling setups are valuable probes of quantum matter with a scope that includes strong correlation features, e.g., the suppression of tunneling in electronic quantum wires [1]; topological phenomena, e.g., zero bias features of Majorana fermions [2, 3]; and even the combination of these, e.g., universal exponents and fractional charge for fractional quantum Hall (FQH) edge modes [4].

Most past work has focused on fermionic systems, considering the electronic conductance in the solid state. Bosonic systems, however, are now attracting much interest. Theoretically, this is in large part due to predictions of novel, symmetry protected topological phases (SPTs) [5, 6] that, as bosonic counterparts of electronic topological insulators, provide topological paradigms without requiring fractional quasiparticles (i.e., they are “nonfractionalized” [7]). Experimentally, much of the interest is due to breakthroughs in ultracold atomic systems, including recent progress with measuring particle conductance [8] (where after the success with fermionic atoms, we anticipate analogous bosonic studies), and with creating (both fermionic and bosonic) topological bandstructures [9, 10].

In this paper, we present two key advances. The first of these is the discovery of a striking degree of universality characterising bosonic nonfractionalised tunneling. To establish this result we consider a set of topological and nontopological setups composed of one dimensional (1D) channels of repulsive bosons (boson “wires”) and/or the 1D edge states of boson integer quantum Hall (bIQH) states [5, 11], the simplest 2D bosonic SPTs protected by particle number conservation. Specifically, we focus on the T-junctions and quantum point contacts (QPCs) in Fig. 1 which are the minimal tunneling setups that admit as subsystems both boson wires and bIQH edge states. The universality we find is in the tunneling con-

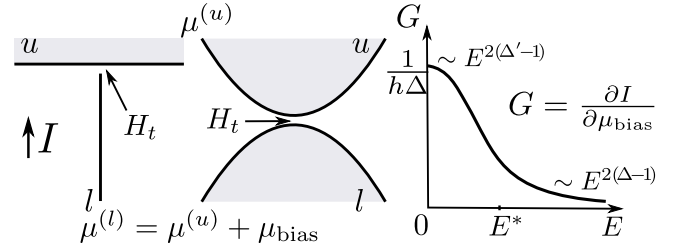


FIG. 1. Sketch of the tunneling setups and the conductance. Left: T-junction. The lower ( $l$ ) subsystem is a half-infinite wire, the upper ( $u$ ) is a wire, or a bIQH edge (or a spurious wire-like edge of a nontopological phase). Middle: QPC with bIQH edge or wire (wire-like) subsystems. For both setups, shading indicates the 2D bulk (absent for wire-like subsystems). Our QPC results also hold for “inverse QPCs” with the 2D bulk located between the two subsystems (i.e., in the blank instead of the shaded area). Right: conductance versus the characteristic energy scale  $E$  (e.g., temperature,  $\mu_{\text{bias}}$ ) valid for all setups on the left and in the middle.  $\Delta < 1$  and  $\Delta' > 1$  are interaction dependent parameters.

ductance  $G$  which, as a function of the energy scale  $E$  (e.g., temperature or chemical potential bias  $\mu_{\text{bias}}$ ), for both geometries and all wire/bIQH edge combinations, is enhanced as  $E$  decreases, with the high and zero energy behavior linked by a universal relation (Fig. 1, right panel).

While the universality we find establishes a key signature of bosonic mesoscopic quantum transport, it also implies challenges in using the conductance to distinguish bIQH states from nontopological gapped phases with spurious wire-like edge modes [12]. Our second innovation, therefore, is to devise schemes whereby tunneling transport can provide clear signatures of bIQH states. The signatures we find include a current imbalance to chemical potential bias ratio quantized as  $\frac{\delta J_{\rho}}{\mu_{\text{bias}}} = \frac{q}{h}$  at low energies in a QPC setup, which holds even though

the clean bIQH edges we consider lack conductance quantization [13, 14]. Here  $q \in \mathbb{Z}$ , which sets the bulk Hall conductivity  $\sigma = \frac{2q}{h}$ , is the topological invariant characterizing the bIQH state. (We define conductivity and conductance as particle current against chemical potential bias, in terms of which the conductance quantum is  $1/h$ .)

Our results assume three main criteria: i) clean wires and bIQH edges; ii) short ranged repulsive interactions in wires; and iii) the system to reach a steady state, after coupling the subsystems, so that an energy-dependent conductance can emerge. These are geared towards ultracold atomic systems, where the first two criteria are naturally met, and the last one is also feasible given the recent progress in quantum transport with neutral atoms [8]. As we will explain, ultracold atoms, at the same time, may provide the currently most promising avenue for realising bIQH states; in particular, as we will point out, a bIQH state with  $q = 1$  can arise with single-component strongly repulsive bosons in the recently observed Harper-Hofstadter bandstructure [9].

Before moving to the details of the analysis, it is instructive to compare our findings to their fermionic [1] and fractionalized [4] analogues and to the  $q=1$  bIQH result in Ref. [14]. The enhancement we find is in stark contrast to the suppression for repulsive (and, for QPCs, also attractive) fermions and fermionic/bosonic FQH edges in the same geometries. The contrast is also salient between hard-core bosons and free fermions: though they are very similar in 1D, tunneling, just at a point, is already away enough from a pure 1D setting for bosonic tunneling enhancement to replace the energy independent free fermion behavior. A similar enhancement, characterizing  $q=1$  bIQH QPCs, was highlighted in Ref. 14. Our results show that this is part of a universal behavior that arises for any  $q$ , including the  $q=0$  case of nontopological wire-like modes or boson wires. Another interesting difference to fermions is in  $\frac{\delta J_\rho}{\mu_{\text{bias}}}$ , which is *not* quantized in fermionic integer quantum Hall QPCs: due to the chirality of the edge modes it is instead given by the (nonuniversal) tunneling conductance.

The paper is organized as follows. In Sec. II we discuss the theoretical description of our systems in terms of Luttinger liquid theory, a language that allows one to discuss boson wires and bIQH edge states on the same footing. In Sec. III we obtain results on the tunneling conductance in the setups of Fig. 1, with the universal high-energy-zero-energy relation as one of our key findings. In Sec. IV we discuss bIQH signatures in the low energy power laws and in the particle and current densities, including the quantization  $\frac{\delta J_\rho}{\mu_{\text{bias}}} = \frac{q}{h}$ . In Sec. V we discuss perspectives for experimentally observing our predictions in ultracold atomic systems, and also highlight a possible route towards bIQH states in terms of single component bosons in Harper-Hofstadter bands. We conclude in Sec. VI.

## II. LUTTINGER LIQUID FORMULATION

We now introduce the framework for the analysis underlying our results. We focus on the low energy physics, i.e., we work below a high energy cutoff  $D_0$  set by the average density  $n_0$  for wires, or the gap for bIQH states. Starting with boson wires, the physics is that of left and right moving density waves. This is captured by a Luttinger liquid description [15] using two bosonic fields  $\phi_{1,2}$ , with  $\phi_1$  encoding the fluctuations  $n_\rho(x) = \frac{1}{2\pi} \partial_x \phi_1(x)$  of the particle density relative to  $n_0$ , and  $\phi_2$  being the phase. The well-known canonical conjugacy of density and phase can be expressed by

$$[\phi_j(x), \phi_k(y)] = i\pi K_{jk}^{-1} \text{sgn}(x-y), \quad K = \sigma_1, \quad (1)$$

where  $\sigma_1$  is the first Pauli matrix.

BIQH edges also support a left and a right moving mode and also form Luttinger liquids with fields  $\phi_{1,2}$  satisfying Eq. (1). The key difference to boson wires is how  $\phi_{1,2}$  enter the particle density: as follows from the Chern-Simons theory (the ‘‘Landau theory’’ of quantum Hall phases [16]), one has  $n_\rho(x) = \frac{1}{2\pi} \mathbf{q}^T \partial_x \boldsymbol{\phi}(x)$  with the integer vector  $\mathbf{q} = (1, q)^T$  [5]. Note that this expression includes the boson wire (nontopological) case as  $q = 0$ .

Taking advantage of these effective theories, we can discuss wire and bIQH systems on the same footing. The Hamiltonian is

$$H_{1D} = \frac{\hbar}{4\pi} \int dx \sum_{jk} \partial_x \phi_j V_{jk} \partial_x \phi_k, \quad (2)$$

where  $V$  is a symmetric positive definite matrix encoding the details of the confinement (or bandstructure in the wire case) and intermode interactions, and which leads to left and right moving modes with velocities  $v_{R/L} = \sqrt{V_{11}V_{22}} \pm V_{12}$ . For wires, in the reflection symmetric case ( $V_{12}=0$ ) [15], we recover the usual Luttinger Hamiltonian with sound velocity  $v = \sqrt{V_{11}V_{22}}$  and Luttinger parameter  $g = \frac{1}{2} \sqrt{\frac{V_{22}}{V_{11}}}$ . For the short range repulsive interactions considered here, we have  $g \geq 1$ , with  $g = 1$  being the hard-core limit; in general,  $g$  is set by  $v$ ,  $n_0$ , and the compressibility  $\kappa$  as  $g = \pi \hbar v n_0^2 \kappa$  [15]. In what follows we will assume  $V_{12} = 0$  for wires, but for wire-like nontopological edges  $V_{12} \neq 0$  will be allowed. In the bIQH cases, without loss of generality, we assume  $q > 0$  and take  $V_{11} < V_{22}$ . In the  $q = 1$  case this choice only implies working away from the special  $V_{11} = V_{22}$  point (reaching which would require careful fine tuning of confinement details); for  $q > 1$  it follows from requiring the dominant edge mode interaction to be through the particle density, which is a reasonable assumption given that microscopic density-density interactions can stabilize bIQH states [11, 17, 18] (see also Sec. V for a discussion).

The commutator Eq. (1) together with the form of  $n_\rho(x)$  establishes the operators creating/removing a given number of particles (or ‘‘charge’’) [16]: from

$[n_\rho(x), e^{-i\lambda^T \phi(y)}] = \mathbf{q}^T K^{-1} \lambda \delta(x-y) e^{-i\lambda^T \phi(y)}$ , we have  $\psi_\lambda^\dagger(x) \sim e^{-i\lambda^T \phi}$  with  $\lambda = (\lambda_1, \lambda_2)^T$ , creating charge  $Q_\lambda = \mathbf{q}^T K^{-1} \lambda$ . From the Chern-Simons theory it follows[16] that  $\lambda_j$  are integers and therefore so is  $Q_\lambda$ ; the theory has no fractional quasiparticles. The Hamiltonian for tunneling between the upper ( $u$ ) and lower ( $l$ ) subsystems can therefore be written as

$$H_t = \sum_{\lambda_{(u)}, \lambda_{(l)}} t_{\lambda_{(u)}, \lambda_{(l)}} e^{i(\lambda_{(l)}^T \phi^{(l)} - \lambda_{(u)}^T \phi^{(u)})} + \text{h.c.}, \quad (3)$$

where the summation is over all tunneling processes involving  $\psi_{\lambda_{(l)}}$  and  $\psi_{\lambda_{(l)}}$  with  $t_{\lambda_{(u)}, \lambda_{(l)}}$  as the tunnel amplitudes, subject to the constraint of charge conservation

$$Q_{\lambda_{(u)}} + Q_{\lambda_{(l)}} = \mathbf{q}_{(u)}^T K_{(u)}^{-1} \lambda_{(u)} + \mathbf{q}_{(l)}^T K_{(l)}^{-1} \lambda_{(l)} = 0, \quad (4)$$

In Eq. (3) the fields are taken at  $x = 0$ , the position of the contact. In T-junctions, for the half-infinite wires ending at  $x=0$  we have the boundary condition  $\phi_1(x \rightarrow 0) = 0$  [19].

In what follows we show that even with this coarse level of detail in the Hamiltonian specification, as befits a phenomenology such as the Luttinger liquid and Chern-Simons theories, one can obtain useful predictions highlighting universal features shared between boson wires and bIQH edge modes and clear observable signatures that distinguish these phases.

### III. TUNNELING CONDUCTANCE

We start by studying how the tunneling behavior changes upon lowering the energy scale  $E$  (e.g. temperature  $k_B T$  or bias  $\mu_{\text{bias}}$ ) of interest. In the theoretical description,  $E$  plays the role of the infrared cutoff. The problem is conveniently analyzed using the renormalization group (RG). This will allow us to identify the most relevant (in the RG sense) term in Eq. (3) which sets the high energy tail and, by providing an emergent boundary condition, the zero energy limit of the tunneling conductance  $G$ . Subsequently, we will study the effect of the leading perturbations that emerge near the zero energy limit, which will allow us to establish the stability of the ground state implied by the zero energy boundary condition (a strong coupling fixed point of the RG) and find the low energy power law conductance corrections.

#### A. High energy regime

Treating  $H_t$  as a perturbation, under an RG elimination of the window  $(\frac{D_0}{b}, D_0)$  below the bare high energy cutoff  $D_0$ , the couplings scale as  $t_{\lambda_{(u)}, \lambda_{(l)}} = t_{\lambda_{(u)}, \lambda_{(l)}}^{(0)} b^{1-\Delta_{\lambda_{(u)}, \lambda_{(l)}}$ , defining the scaling dimension  $\Delta_{\lambda_{(u)}, \lambda_{(l)}}$ . Since  $b > 1$ , the dominant (most relevant in the RG sense) process is the one with the smallest  $\Delta_{\lambda_{(u)}, \lambda_{(l)}}$ .

Following standard steps [13, 15, 16] we find  $\Delta_{\lambda_{(u)}, \lambda_{(l)}} = \Delta_{\lambda_{(u)}} + \Delta_{\lambda_{(l)}}$  where  $\Delta_\lambda$  is the scaling dimension of one  $e^{\pm i\lambda^T \phi}$  factor in Eq. (3). For the end of a half infinite wire  $\Delta_\lambda = \frac{Q_\lambda^2}{2g}$ , while for a bIQH edge or the bulk of a wire  $\Delta_\lambda = \frac{1}{2} \sqrt{\frac{V_{22}}{V_{11}}} \lambda_1^2 + \frac{1}{2} \sqrt{\frac{V_{11}}{V_{22}}} (Q_\lambda - q\lambda_1)^2$ . We find that the dominant term corresponds to  $\lambda_{(l)} = -\lambda_{(u)} = (0, \pm 1)^T$ , independent of  $q$ . It describes the single particle ( $|Q_{\lambda_{(u,l)}}| = 1$ ) process

$$H_{t_0} = t_0 e^{i(\phi_2^{(l)} - \phi_2^{(u)})} + \text{h.c.} \quad (5)$$

with scaling dimension  $\Delta = \frac{1}{2} \sqrt{\frac{V_{11}^{(u)}}{V_{22}^{(u)}}} + \frac{1}{2g_{(l)}}$  for T-junctions, and  $\Delta = \frac{1}{2} \sqrt{\frac{V_{11}^{(u)}}{V_{22}^{(u)}}} + \frac{1}{2} \sqrt{\frac{V_{11}^{(l)}}{V_{22}^{(l)}}}$  for QPCs. Importantly,  $\Delta < 1$ : the tunnel coupling increases under RG as the energy scale is lowered (it is RG relevant). This increase translates to the high energy tail of the conductance enhancement in Fig. 1,  $G \sim E^{2(\Delta-1)}$ , which is valid for  $E \gg E^* = D_0 \left( \frac{|t_0^{(0)}|}{D_0} \right)^{\frac{1}{1-\Delta}}$  with  $E^*$  being the characteristic energy scale for the breakdown of the weak coupling (small  $t_0$ ) description. The power laws here and below hold when  $E$  is much larger than other infrared energy scales (e.g.,  $\mu_{\text{bias}} \gg k_B T$  for  $E = \mu_{\text{bias}}$  or  $k_B T \gg \mu_{\text{bias}}$  for  $E = k_B T$ ).

#### B. Zero energy limit and universality

That the high energy tail of  $G$  is of the same schematic form for all junctions is a consequence of Fermi's golden rule[1]. However, it is less expected that, in all cases,  $\Delta$  also sets the zero energy limit  $G(E \rightarrow 0) = \frac{1}{h\Delta}$ . We now establish this universal result, assuming that the zero energy physics is governed by  $H_{t_0}$  at strong coupling.

The key observation is that with  $H_{t_0}$  only, we can relate our setups to a junction between two half-infinite wires. In that case,  $\Delta = \frac{1}{2g_{(u)}} + \frac{1}{2g_{(l)}}$  and  $G = \frac{1}{h\Delta}$  follows from results [1, 20] on fermion Luttinger liquids with  $\Delta < 1$ , because the wire Hamiltonian, the tunnel coupling, and the correlation functions (current-current correlators calculated, e.g., in the lower subsystem) underlying  $G$  at zero energy are defined by the same expressions in the boson and (bosonized) fermion cases. The basis of mapping T-junctions and QPCs to a junction of two half-infinite wires is the so-called unfolding (see Fig. 2): in this description the left and right moving modes of the two half-infinite wires are joined up at  $x = 0$  (are ‘‘unfolded’’)[19] so that the junction consists of two right movers  $\phi_R^{(u,l)}$  on the full line, coupled at  $x = 0$ . In terms of these, the wire Hamiltonians are  $H_{1D}^{(u,l)} = \frac{\hbar v^{(u,l)}}{4\pi} \int dx (\partial_x \phi_R^{(u,l)})^2$ , with  $[\phi_R^{(j)}(x), \phi_R^{(k)}(y)] = i\pi \delta_{jk} \text{sgn}(x-y)$ , and the coupling is  $H_{t_0} = t_0 \exp[i(\sqrt{2\Delta_{(u)}} \phi_R^{(u)} - \sqrt{2\Delta_{(l)}} \phi_R^{(l)})] + \text{h.c.}$

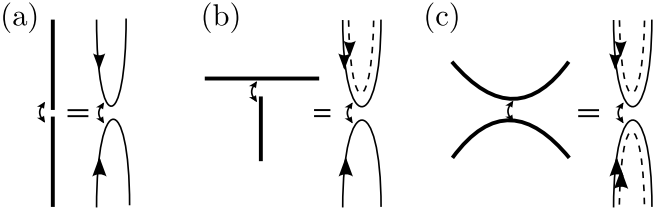


FIG. 2. Mapping T-junctions and QPCs to linear junctions. Diagram (a) shows the unfolding for linear junctions; diagrams (b) and (c) show the transformations for T-junctions and QPCs. Dashed lines indicate the decoupled modes.

Turning now to T-junctions, using that left and right movers are eigenmodes of  $H_{1D}$  [Eq. (2)] and thus are coupled only at  $x = 0$ , one can flip the left mover component on the upper (bIQH or wire) side and rescale  $x$  to formally treat  $\phi_2^{(u)}$  in Eq. (5) as the combination of two right movers with the same velocities. One can then introduce this combination as a new right-mover field  $\phi_{R1}^{(u)}$  (and the orthogonal combination as  $\phi_{R2}^{(u)}$ ). In terms of these, the upper side Hamiltonian is  $H_{1D}^{(u)} = \frac{\hbar v}{4\pi} \sum_{\alpha=1,2} \int dx (\partial_x \phi_{R\alpha}^{(u)})^2$ , with  $[\phi_{R\alpha}^{(u)}(x), \phi_{R\beta}^{(u)}(y)] = i\pi\delta_{\alpha\beta} \text{sgn}(x-y)$ , and the coupling is  $H_{t_0} = t_0 \exp[i(\sqrt{2\Delta_{(u)}}\phi_{R1}^{(u)} - \sqrt{2\Delta_{(l)}}\phi_R^{(l)})] + \text{h.c}$  where  $\phi_R^{(l)}$  is the unfolded right mover on the lower (half-infinite wire) side. Notice that the field  $\phi_{R2}^{(u)}$  decouples. From the perspective of the lower side, the upper side (through  $\phi_{R1}^{(u)}$ ) acts effectively as another half-infinite wire with Luttinger parameter set by  $\Delta_{(u)}$ . Calculating the zero energy conductance from the lower side current-current correlators, we therefore find  $G = \frac{1}{\hbar\Delta}$  with  $\Delta$  for the T-junction. Of course, the same conductance should be found if it is calculated from the upper side correlators. This immediately shows that  $G = \frac{1}{\hbar\Delta}$  also holds for QPCs: performing now the same steps on the lower side, from the perspective of the upper side a QPC looks the same as a T-junction, with the lower side Luttinger parameter set by  $\Delta_{(l)}$ . This completes our proof that  $G = \frac{1}{\hbar\Delta}$  holds for all junctions at zero energy.

### C. Low energy power laws

To complete the analysis of the low energy behavior we calculate the dominant scaling dimension  $\Delta'$  near strong coupling ( $t_0 \rightarrow \infty$ ). If  $\Delta' > 1$  is found, the strong coupling fixed point is stable and the conductance displays low energy power laws given by  $\delta G \sim E^{2(\Delta'-1)}$  (for  $E \ll E^*$ ). We obtain  $\Delta'$  using a method analogous to Refs. [19, 21]: we identify the spectrum of allowed perturbations via the  $x \rightarrow 0$  limit of charge conserving operators subject to the boundary condition implied by  $t_0 \rightarrow \infty$ . From this we obtain  $\Delta'$  as the smallest in the consequent spectrum of scaling dimensions.

Starting with linear junctions, for completeness, we find  $\Delta' = \frac{1}{\Delta}$  similarly to fermionic results[1]. For T-junctions we also find  $\Delta' = \frac{1}{\Delta}$  unless  $\gamma = \sqrt{\frac{V_{22}^{(u)}}{V_{11}^{(u)}}}$  satisfies  $g_{(l)} - \sqrt{g_{(l)}^2 - q^2} < \gamma < g_{(l)} + \sqrt{g_{(l)}^2 - q^2}$  (for  $g_{(l)} > q$ ), in which case  $\Delta' = \frac{q^2 + \gamma^2 + 2\gamma g_{(l)}}{2(\gamma + g_{(l)})}$ . For QPCs, taking the same interactions on the upper and lower sides for brevity, we find  $\Delta' = \frac{1}{\Delta}$  for  $\frac{1}{q} < \sqrt{\frac{V_{11}}{V_{22}}} < 1$  and  $\Delta' = \frac{3 + \Delta^2 q^2}{4\Delta}$  for  $\sqrt{\frac{V_{11}}{V_{22}}} < \frac{1}{q}$  (with  $\frac{1}{q} = \infty$  for  $q = 0$ ). Importantly, for the interactions considered here  $\Delta' > 1$ : the strong coupling fixed point is stable. (This also holds for QPCs with unequal upper and lower side interactions.) It is interesting to note that for boson wire T-junctions and QPCs characterised by  $g_{(u)} = g_{(l)}$ , both the zero temperature conductance and the low energy power laws agree with those of multi-junctions of semi-infinite boson wires [22] and fermionic topological Kondo systems [23, 24] with the latter related to the former by bosonization [24].

### D. Comparison to fermion Luttinger liquids

As we emphasized in the Introduction, the behavior we find is in stark contrast to the fermionic case. Given that 1D fermions also admit a Luttinger liquid description, one may wonder where this striking difference originates. It is firstly noticeable that, though for fermions  $\mathbf{q}$  and  $K$  are the same as for boson wires, fermions with repulsive, attractive, or no interactions have  $g < 1$ ,  $g > 1$ , or  $g = 1$ , respectively. However, as the fermionic comparisons in the Introduction indicate, this difference in interaction parameters is not the critical reason for the markedly different bosonic tunneling behavior. The key difference is in the tunneling terms where for fermionic single particle processes exchange statistics forces  $\lambda_{1,(u/l)}$  in Eq. (3) to be half-integer [16]. This renders  $H_{t_0}$  (which has  $\lambda_{1,(u/l)} = 0$ ) invalid for fermions, resulting in markedly different behavior.

## IV. BIQH SIGNATURES

The universality of the conductance, while a striking signature of bosonic nonfractionalised tunneling, implies challenges for detecting bIQH states via conductance measurements. Indeed, the only feature of  $G$  that we found to be capable of detecting the topological invariant  $q$  is the low energy power law, which becomes  $q$  dependent in the cases with  $\Delta' \neq 1/\Delta$ . Though challenging, a protocol for measuring  $q$  via  $G$  is as follows: if  $\Delta' \neq \Delta$  is measured,  $q$  can be extracted from our expression for  $\Delta'$  using data on  $\Delta$  for a symmetric QPC, or on both  $\Delta$  and  $g_{(l)}$  for a T-junction. Here  $\Delta$  can be determined by  $G$  at zero or high energy, and  $g_{(l)}$  of the boson wire can be

obtained [15] either via data on the microscopic interactions or measurements of density correlations. Note that in T-junctions, the lower side boson wires can always be tuned to reach  $\Delta' \neq \frac{1}{\Delta}$  by increasing  $g_{(l)}$  (i.e., weakening the repulsive interactions). While a  $q$  dependent conductance power law is already a well defined bIQH feature, as we explain below, tunneling setups also provide signatures that are more qualitative and experimentally more feasible.

Intuitively, by solid-state analogy, the Hall resistance  $R_H$  in a four terminal arrangement would be a natural candidate. With cold atoms, however, measuring the Hall voltage  $V_H$ , and having locally thermalized Hall current carrying edge modes for its clear interpretation may be challenging. Surprisingly, even these issues aside,  $R_H$  is of limited utility in tunneling setups, as it turns out to vanish for a clean edge for  $T \rightarrow 0$  due to  $H_{t_0}$  remaining the only coupling. This can be seen as follows: firstly, due to the commutation relation Eq. (1),  $H_{t_0}$  [Eq. (5)] tunnels only into mode  $\phi_1$ , thus measuring  $V_H$  via a tunnel contact probes  $\mu_1$  (the chemical potential for  $\phi_1$ ) and the current for  $\phi_2$  satisfies  $\partial_x J_2 = 0$  in a steady state. Secondly, in a thermal state  $J_2 = \mu_1/h$  [25]. Therefore,  $\mu_1$  is constant along the whole edge, implying that  $V_H$  and thus  $R_H$  vanishes. In fact, the observation that  $H_{t_0}$  tunnels only to  $\phi_1$  explains all the similarities between bIQH edges and boson wires we encountered: if tunnel junctions are the only sources of data (e.g., via  $G$  or as voltage probes), apart from small corrections due to the perturbations near strong coupling, one effectively probes only the  $\phi_1$  (i.e., the first) component of  $\mathbf{q}$  which is  $q$  independent.

A bIQH state being a Hall system, a transport current should however, regardless of details, induce some imbalance in charge density  $\delta n_\rho$  and current  $\delta J_\rho$  between opposite edges which should be  $q$  dependent. Not requiring voltage measurement, nor thermalized edges, these observables are natural replacements to  $R_H$  for cold atomic settings. As we now show, tunneling setups are particularly advantageous for measuring  $\delta n_\rho$  and  $\delta J_\rho$  as nonuniversalities due to intermode interactions can be eliminated. The key quantities are summarised in Fig. 3. The equations of motion relate the currents  $\mathbf{J}$  and densities  $\mathbf{n} = \frac{\partial_x \phi}{2\pi}$  of the two edge modes as  $\mathbf{J} = K^{-1}V\mathbf{n}$  [25]. When  $H_{t_0}$  dominates, in a steady state we have  $\partial_x J_1 = I\delta(x)$  and  $\partial_x J_2 = 0$  up to small corrections, where  $I$  is the particle current between the subsystems. Integrating across the contact we find

$$\frac{\delta n_\rho}{I} = \frac{1 + q\sqrt{\frac{V_{11}}{V_{22}}}}{2v_R} - \frac{1 - q\sqrt{\frac{V_{11}}{V_{22}}}}{2v_L} \quad (6)$$

for the charge imbalance  $\delta n_\rho = n_{\rho,R} - n_{\rho,L}$  between the right and the left edge segments. The deviation of  $\delta n_\rho$  from  $\frac{1}{2}(v_R^{-1} - v_L^{-1})$  is a qualitative bIQH signature. In the special case  $\frac{V_{11}}{V_{22}} = q^{-2}$ , the left moving mode, akin to an edge dipole mode, carries no particle density. In this case, instead of the comparison to  $\frac{1}{2}(v_R^{-1} - v_L^{-1})$ , a clear

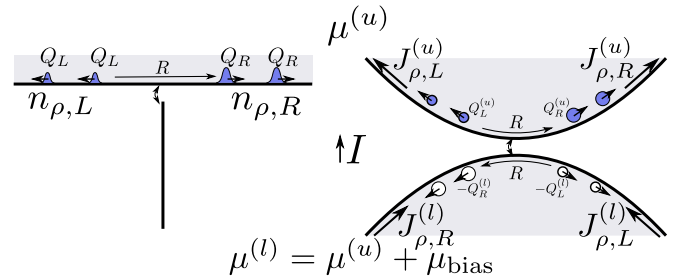


FIG. 3. Charge densities  $n_{\rho,R/L}$  and currents  $J_{\rho,R/L}$  underlying predictions Eqs. (6) and (7). We measure  $n_{\rho,R/L}$  and  $J_{\rho,R/L}$  relative to their equilibrium values at  $\mu^{(u/l)}$ . The right moving direction ( $R$  arrows) is taken relative to the Hall state chirality. For  $\delta J_\rho = J_{\rho,R}^{(u)} - J_{\rho,L}^{(l)}$  we find  $\frac{\delta J_\rho}{\mu_{\text{bias}}} = \frac{q}{h}$  at low energies. The intuitive picture is in terms of charge  $Q_{R/L}^{(u/l)}$  solitons.

bIQH signature is provided by the density disturbances propagating only to the right.

Eq. (6) can also be derived using an intuitive picture:

first, one can show that  $Q_{R/L} = \frac{1 \pm q\sqrt{\frac{V_{11}}{V_{22}}}}{2}$  is the charge of the right or the left moving soliton that arises due to injecting a unit charge into  $\phi_1$  (i.e., a single  $H_{t_0}$  transport event). Then, identifying  $I$  as the injection rate, we have  $n_{\rho,R/L} = \frac{Q_{R/L}}{v_{R/L}} I$  as the charge density to the right/left of the contact, thus explaining Eq. (6).

In a T-junction we have  $\sqrt{\frac{V_{11}}{V_{22}}} = 2\Delta - g_{(l)}^{-1}$  thus  $q$  can be extracted from  $\frac{\delta n_\rho}{I}$  using  $v_{R/L}$ ,  $\Delta$ , and  $g_{(l)}$ . Measuring  $\frac{n_{\rho,R/L}^{(u/l)}}{I}$ ,  $v_{R/L}$  and  $\Delta$ , one can also extract  $q$  using a QPC. QPCs, however, also provide a more elegant option via the current  $J_{\rho,L/R} = v_{L/R} n_{\rho,L/R}$ . In terms of  $\delta J_\rho = J_{\rho,R}^{(u)} - J_{\rho,L}^{(l)}$  we find  $\frac{\delta J_\rho}{I} = q\Delta$ . At low temperatures and in the linear regime one has  $I = G\mu_{\text{bias}} = \frac{\mu_{\text{bias}}}{h\Delta}$  up to small power law corrections, which gives

$$\frac{\delta J_\rho}{\mu_{\text{bias}}} = \frac{q}{h}, \quad (7)$$

the quantization announced in the Introduction. Note that Eq. (7) does not require a symmetric QPC.

## V. EXPERIMENTAL PERSPECTIVES

We now turn to some considerations on testing our predictions in ultracold atomic systems, discussing routes towards realising boson wires and bIQH states and towards nonequilibrium measurements of  $G$ ,  $\delta n_\rho$  and  $\delta J_\rho$ .

### A. Boson wires and a Harper–Hofstadter-model based bIQH state

The boson wires for our tunneling setups can be realized by strongly repulsive bosonic atoms in 1D optical

confinement, e.g., by suitably adapting the wire arrays in Ref. 26, or, in an approach particularly suited for our geometries, by utilizing the holographic optical traps of Ref. 27. A realization of a bIQH state was originally proposed [11] and later numerically demonstrated [17] for two-component bosons in the lowest Landau level. Optical lattice systems of current experiments [9, 10], however, may provide a bIQH option already with single-component particles. This is easiest seen for  $q=1$ :  $K=\sigma_1$  and  $\mathbf{q}=(1,1)$  is in the composite fermion series [28] for bosons in a periodic potential [18, 29], corresponding to the case when composite fermions fill bands of total Chern number  $|C|=2$  (as  $K$  is  $2\times 2$ ) with the attachment of a single flux quantum per particle [transforming  $K_0=-\mathbb{1}_2$  of filled bands of Chern number  $-2$  to  $K=-\mathbb{1}_2+(\frac{1}{1}\frac{1}{1})=\sigma_1$ ] [30]. Interestingly, this state was numerically observed [18] in an FQH study of hard-core bosons in the Harper-Hofstadter bandstructure, where a ground state captured by a composite fermion trial wavefunction with the above attached flux and composite fermion Chern number (as extracted from the many body Chern number using the results of Ref. 29) was demonstrated, thereby providing the earliest instance of a bIQH phase (see also Ref. 31). FQH results on the same model also suggest that well-defined edge modes may be possible under realistic conditions [32]. The cold atom realisation of the Harper-Hofstadter bandstructure [9] is thus particularly promising from the bIQH perspective. Recently, bIQH states were also predicted in optical flux lattices [33] and in models with correlated hopping [34].

### B. Transport observables via steady state measurements

Testing our transport predictions requires a steady state but only needs initial states to be thermal. This suggests approaches inspired by Ref. 8: with  $H_t$  off, one prepares the upper and lower subsystems initially decoupled, in thermal states with a common temperature, and with  $\mu_{\text{bias}}$  between them. Then, one turns  $H_t$  on to let the full system evolve. In the steady state, we expect [35]

$\frac{\partial I}{\partial \mu_{\text{bias}}} = G[E = \max(k_B T, \mu_{\text{bias}})]$  when one of the two energy scales  $\{k_B T, \mu_{\text{bias}}\}$  is much larger than the other. The imbalance  $\delta n_\rho$  can be measured by standard methods for detecting particle density. The recent demonstration of detecting local lattice currents [36] also provides prospects for observing  $\delta J_\rho$ .

## VI. CONCLUSION

In conclusion, we have shown that bosonic nonfractionalized tunneling displays emblematic universal features. Tunneling is enhanced upon lowering the energy, with the high energy power law  $G \sim E^{2(\Delta-1)}$  tied to the zero energy limit  $G = \frac{1}{h\Delta}$ . This universal relation holds for all the interactions and geometries we considered, and is valid regardless of the topological nature of the modes. While the tunneling conductance of wires and bIQH edges thus share remarkable universal features, we have also shown how the characteristic topological invariant  $q$  of the bIQH states is revealed in tunneling via the low energy power laws  $\delta G \sim E^{2(\Delta'-1)}$  (in the regimes that  $\Delta' \neq 1/\Delta$ ), the charge imbalance Eq. (6), and the quantization  $\frac{\delta J_\rho}{\mu_{\text{bias}}} = \frac{q}{h}$ . Our predictions, requiring only initial states to be thermal, are well suited for cold atom experiments. Testing our wire results would be a natural first direction in exploring bosonic mesoscopic quantum transport. With the rapid experimental progress towards bIQH host bandstructures, using our results to detect bIQH states is another intriguing perspective. Our work can also stimulate theoretical explorations of generalizations of our results to multimode (e.g., multi-species) wires and SPTs beyond bIQH states.

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